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Nonlinear Response and Fatigue Life of Isotropic Panels Subjected to Nonwhite Noise

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In stochastic structural dynamics, the majority of the analyses have dealt with linear structures under stationary Gaussian and band-limited white-noise excitation. Recorded aircraft acoustic pressure fluctuations have shown 1) the high levels of acoustic excitation that can drive the surface panels to nonlinear large deflection response and 2) the nonwhite power spectral density characteristic that can affect panel response and fatigue life. This paper presents the nonlinear response and fatigue life estimation of isotropic panels subjected to nonwhite acoustic excitation. An efficient finite element time-domain modal formulation was employed to determine the panel response. The Palmgren–Miner cumulative damage theory in combination with the rainflow counting cycles method was used to estimate the panel fatigue life. To assess the effects of nonwhite power-spectral-density characteristics, an equivalent band-limited white-noise excitation, which has the same acoustic power within the bandwidth as the recorded data, was simulated. Nonlinear response and fatigue life to recorded and simulated data were determined for comparison. Results show that the actual flight data with nonwhite power spectral density can yield higher stress characteristics and shorter fatigue life than the corresponding equivalent white noise.

Nomenclature

D	=	damage
E	=	Young's modulus
$E[\,]$	=	expected value
f_c	=	cutoff frequency
f_0	=	mean rate of occurrence of peaks
G	=	shear modulus
$[I]$	=	identity matrix
K	=	material constant
$[K]$	=	system stiffness matrices
$[\bar{K}_L]$	=	linear stiffness modal matrix
$[\bar{K}_{qq}]$	=	nonlinear stiffness modal matrix
$[M]$, $[\bar{M}]$	=	system and modal mass matrices
n, N	=	actual and total number of cycles
$\{\boldsymbol{P}\}$, $\{\bar{\boldsymbol{P}}\}$	=	system and modal force vectors
$p()$	=	probability density function
$\{\boldsymbol{q}\}$	=	modal coordinate vector
S	=	strain, stress
T	=	fatigue life
u, v	=	in-plane displacements
β	=	material constant
Γ	=	gamma function
Δt	=	integration time step
ν	=	Poisson's ratio
ξ	=	damping ratio
ρ	=	mass density
σ_s	=	standard deviation
$[\phi]$	=	system eigenvector matrix
ω	=	natural frequency

Subscripts

b	=	bending
m	=	membrane
Nm	=	stiffness matrices due to $\{N_m\}$

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Introduction

TRADITIONAL sonic fatigue design and analysis methods for military aircraft[1,2] are based on linear structural theory and stationary Gaussian band-limited white-noise excitation (WN). Limited available recorded acoustic pressure fluctuations, however, have shown (Fig. 1) the characteristics of 1) the extremely high levels of excitation, 2) a nearly Gaussian distribution of the probability density function (PDF), and 3) a highly nonwhite characteristic of the power spectral density (PSD).

The high acoustic levels of excitation can drive the surface panels to nonlinear large deflection response. Vaicaitis and his coworkers[3,4] have applied the Galerkin's method to governing partial differential equations (PDE/Galerkin) in conjunction with the Monte Carlo simulation for the prediction of nonlinear response of isotropic and composite panels subjected to acoustic and thermal loads. Lee[5] has used the PDE/Galerkin method in conjunction with the statistical or equivalent-linearization[6] (EL) technique to study the nonlinear random response of vibrating isotropic plates. The use of the PDE/Galerkin method, however, limits its applicability to simple panels of rectangular shape, simple boundary conditions, and symmetric orthotropic composite panels. Extensions of the finite element (FE) methods in conjunction with the time-domain simulation to predict nonlinear response of composite and isotropic panels were reported by Robinson[7] and Green and Killey.[8] The equations of motion in Refs. 7 and 8 were formulated in the structural node degrees of freedom (DOF), and this turned out to be computationally costly because of the large number of structural node DOF and the updating of the nonlinear stiffness matrix at each integration time step. An efficient FE modal formulation using rectangular elements for nonlinear response of isotropic plates under acoustic excitation was developed by Locke[9] in conjunction with the EL technique and an iterative scheme. The FE/EL procedure was

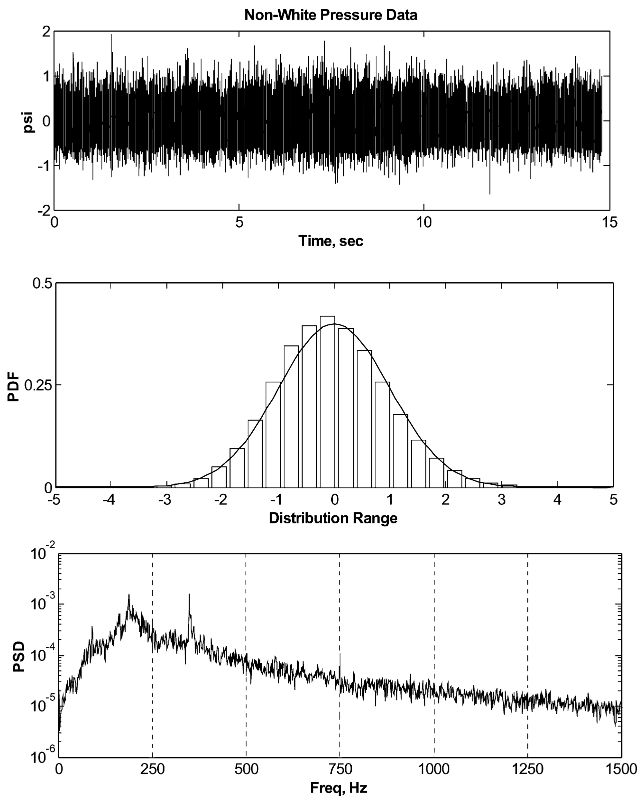

Fig. 1 Time histories, PDF, and PSD of recorded flight data.

further extended to composite panels of arbitrary shape using triangular elements by Mei and Chen.[10] The FE/EL method, however, did not give accurate predictions for snap-through and nonlinear large deflection random motions. The inaccuracy was because the random response considers only one of the two coexisting thermal postbuckling positions, and the deflection PDFs for those two motions are highly non-Gaussian. A versatile procedure to determine the nonlinear large deflection response of panels subjected to high flight acoustic levels was thus the use of the recently developed efficient FE time-domain modal formulation.[11,12] Reviews of large deflection analysis in sonic fatigue design were given by Mei and Wolfe,[13] Benaroya and Rebak,[14] Clarkson,[15] and Vaicaitis.[16]

As discussed earlier, nonlinear response of surface panels to acoustic loads has been limited to stationary Gaussian and band-limited white noise. This paper presents for the first time the nonlinear response and fatigue-life estimation for isotropic panels subjected to the nonwhite pressure fluctuations. One of the objectives of this study is to assess the effects of excitation of nonwhite PSD on panel response and on fatigue life as compared with the traditional analysis of white-noise assumption. Limited results of this study showed that the actual flight pressure fluctuations of nonwhite PSD yield higher stresses and shorter fatigue life than the traditional band-limited white-noise excitation.

Random Surface Panel Pressure

As summarized by Crandall and Zhu,[17] the three important aerospace problems of random vibration are the following: 1) buffeting of aircraft by atmospheric turbulence, 2) sonic fatigue of aircraft and spacecraft surface panels as a result of jet noise impingement or boundary-layer pressure fluctuations, and 3) the reliability of payloads in rocket-propelled vehicles. The present work is going to analyze the second type of problem. The recorded flight pressure fluctuation is presented first, followed by the simulation of the equivalent band-limited white noise (EWN).

Flight Acoustic Pressure

The data set shown in Fig. 1 of recorded in-flight acoustic pressure fluctuations has exhibited the characteristics of 1) extremely high levels of excitation, 2) a nearly Gaussian distribution of the PDF, and 3) a highly nonwhite (nonflat) characteristic of the PSD (psi^2/Hz). The microphone data file corresponds to an aircraft with full afterburners in takeoff configuration. The takeoff consists of three consecutive maneuverings: 1) rolling (4.95 s), 2) rotation (4.9 s), and 3) gear-up (4.95 s), giving a total duration of 14.80 s. The high acoustic levels of excitation shown in the time-history plot brought the panel response to the nonlinear large deflection region. The PSD plot gave approximately a variation of 10^2 psi^2/Hz or 20 dB sound pressure level (SPL) between the maximum and minimum values. In addition, the nonwhite PSD characteristics caused the higher modes in the neighborhood of 200 Hz to respond more than the lower modes to the total stress as compared to the traditional band-limited white-noise excitation. The higher modes yield higher stress and vibrate at higher frequencies than the lower modes. This can lead to a shorter fatigue life as compared with its corresponding EWN.

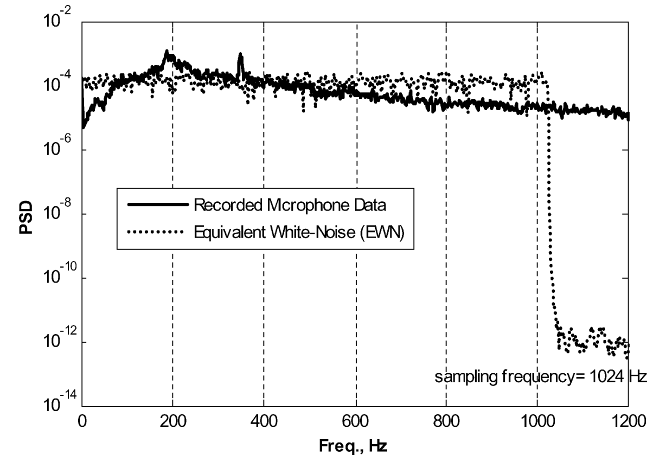

Fig. 2 PSD of EWN and recorded aircraft data.

Equivalent Band-Limited White Noise

To assess the effects of nonwhite PSD excitation, nonlinear response and fatigue life for the same panel subjected to the stationary Gaussian white noise were obtained. An equivalent band-limited white noise (EWN) with cutoff frequency of 1024 Hz was simulated. Figure 2 shows the PSD of the EWN with the same acoustic power within the 1024-Hz bandwidth as its corresponding recorded data. The SPL of the pressure fluctuation was 131.91 dB (162.01 dB overall). The PSD of the recorded acoustic pressure data were also plotted for comparison. In the present case, only one set of microphone data was available for analysis. However, if more data sets were available it would be preferred to follow a Monte Carlo numerical procedure. The Monte Carlo procedure would reduce the uncertainty in fatigue-life estimation because it is calculated from a large number of samples for the average of the ensemble.

Finite Element Modal Formulation and Panel Response

The versatile finite element time-domain modal formulation is employed to determine the time histories of panel maximum deflection and stress. The system equations of motion for an isotropic or symmetrically laminated composite panel can be expressed as a set of coupled nonlinear modal equations[11]:

$$[\bar{M}]\{\ddot{q}\} + 2\xi_r\omega_r\bar{M}_r[I]\{\dot{q}\} + ([\bar{K}_L] + [\bar{K}_{qq}])\{q\} = \{\bar{P}\} \tag{1}$$

in which the diagonal modal mass and linear stiffness matrices are

$$[\bar{M}] = [\phi]^T[M_b][\phi] = \bar{M}_r[I] \tag{2}$$

$$[\bar{K}_L] = [\phi]^T[K][\phi] \tag{3}$$

and the nonlinear cubic term is

$$[\bar{K}_{qq}]\{q\} = [\phi]^T \sum_{r=1}^{n}\sum_{s=1}^{n} q_r(t)q_s(t)\Big([K_{2b}]^{(rs)} - [K_{2\mathrm{Nm}}]^{(rs)} - [K_{1\mathrm{bm}}]^{(r)}[K_m]^{-1}[K_{1\mathrm{mb}}]^{(s)}\Big)[\phi]\{q\} \tag{4}$$

and the modal random load vector is

$$\{\bar{P}\} = [\phi]^T \{P_b(t)\} \tag{5}$$

The random modal amplitude $\{q\}$ was determined from Eq. (1) using the fourth-order Runge–Kutta numerical integration scheme for known random load of white or nonwhite PSD. The efficiencies in using the modal equations presented in Eq. (1) are the following: 1) the number of modal equations is small, and 2) the nonlinear modal stiffness matrices $[K_{2b}]^{(rs)}$, $[K_{2\mathrm{Nm}}]^{(rs)}$, and $[K_{1\mathrm{bm}}]^{(r)}$ are constant; therefore, there is no need to update the nonlinear cubic term at each time step. Once the modal amplitude $\{q\}$ was determined, time histories of the panel deflection and maximum stress/strain are calculated and their corresponding statistics obtained. The readers are referred to Refs. 11 and 12 for details of the efficient finite element modal formulation.

Fatigue-Life Estimation

The method employed for fatigue-life analysis is the well-known and widely used Palmgren–Miner[18] cumulative damage theory

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} = 1.0 \tag{6}$$

where n_i and N_i are the actual number of cycles at a given stress level and the number of total cycles at which failure occurs at the same stress level. The stress/strain vs the number of cycles to failure (S-N) curve normally takes the form

$$N = K/S^{\beta} \tag{7}$$

where the material constants K and β are determined experimentally. Because of the large scatter of fatigue data, those constants were then modeled as random variables leading to a stochastic fatigue prediction model. In this paper, for simplicity in fatigue-life calculation the expected values $E[K]$ and $E[\beta]$ were considered as deterministic quantities.

Stress response under random surface panel pressures is also random, and the Palmgren–Miner theory is rewritten as

$$E[D(t)] = \frac{1}{K}\int_0^{\infty} p(s)s^{\beta}\,\mathrm{d}s$$

$$= E\left[\frac{1}{K}\sum p(s)s^{\beta}\right] = E\left[\frac{1}{K}\sum s_k^{\beta}\right] \tag{8}$$

where $p(S)$ is the stress/strain amplitude or peak probability distribution function (PPDF). Based on Eq. (8), the simplest fatigue-life estimate is

$$T^f = 1/E[D(t)] \tag{9}$$

Some of the cycle counting methods to predict $p(S)$ are 1) the peak counting method, 2) the range counting method, and 3) the rainflow counting cycles (RFC) method. Langley and McWilliam[19] showed that the first two methods give similar results for narrowband processes, but quite different for a wideband process. The RFC method uses a specific cycle counting scheme to account for effective stress ranges and identified stress cycles related to closed hysteresis loops in the cycle stress-strain curves. The validity of the RFC method was studied in detail by Dowling,[20] where the accuracy of fatigue-life predictors, which were based on eight commonly used cycle counting methods, were investigated. The conclusion of Dowling's confirmation experiment was "... the counting of all closed hysteresis loops as cycles by means of the rainflow counting method allows accurate life predictions. The use of any method of cycle counting other than range pair or rainflow methods can result in inconsistencies and gross differences between predicted and actual fatigue lives." In addition, Bishop and Sherratt[21] showed that fatigue life of wideband Gaussian signal using the RFC yields to the most realistic estimates. For comparison purposes, the RFC method is applied to the stress amplitude under the nonwhite flight data and corresponding EWN.

Results and Discussion

Finite Element Validation

The nonlinear finite element modal equations expressed in Eq. (1) are general in the sense that they are applicable for beam, rectangular, and triangular isotropic and orthotropic plate elements. The finite element employed in the present study is the C^1-conforming rectangular plate element. The linear stiffness and mass matrices were developed by Bogner-Fox-Schmit[22] (BFS), and the BFS element has a total of 24 DOF, 16 bending DOF, and eight in-plane DOF.

Validation of the nonlinear modal formulation consisted of two parts: 1) comparison of the nonlinear coefficients between the PDE/Galerkin multimodal Duffing equations and the corresponding finite element cubic terms, $[\bar{K}_{qq}]$ and 2) by comparison with experimental results of a clamped-clamped beam.[23] For the analytical expression of the multimodal Duffing equation coefficients, the readers are referred to Dowell.[24] A mesh-convergence study was performed on a quarter-plate model for the nonlinear coefficients of the modes (1,1) and (3,1). The geometry and material properties of the simply supported panel were $14 \times 10 \times 0.040$ in. ($35.56 \times 25.4 \times 0.10$ cm), modulus of elasticity $E = 10.587$ Msi (73 GPa), Poisson ratio $\nu = 0.3$, mass density $\rho = 2.5845 \times 10^{-4}$ lb-s^2/in.^{4} (2750 kg/m^3), and proportional damping ratio of $\xi_r\omega_r = \xi_s\omega_s$ with $\xi_1 = 0.02$. The values of the coefficient corresponding to the cubic modal displacement $\{q\}$ for the corresponding (i, j) mode of the two coupled modal equations for different mesh sizes are tabulated in Table 1.

From Table 1 it is observed that for the two-mode solution all of the nonzero coefficients were within 1% difference except for q_{31}^3, which is 1.2%. The "zero" coefficients are negligible.

The second part of the validation is by comparison with experimental results of a clamped-clamped beam.[23] The rms displacement for four excitation cases are presented in Table 2 in comparison to the experimental results. The rms values are in good agreement with experimental values for all levels of excitation. In Table 3, the rms values of surface strain are compared to experiment values. Again, the values compared very well. In all cases, the difference compared to experimental results was less than 10%. This difference was considered as good given the complexity of the problem and experimental procedure.

Fatigue-Life Model Validation

Fatigue-life validation consists of the verification of Eqs. (8) and (9) for narrowband signals and for the reproduction of fatigue-life estimate for a complex process with broadband characteristics

Table 1 Comparison of nonlinear coefficients for 14 × 10 × 0.04-in. S-S panel

Modal	q_{11}^3	$q_{11}^2 q_{31}$	$q_{31}^2 q_{11}$	q_{31}^3
		PDE/Galerkin		
1st eq.	$1.2966E12$	$-8.2707E11$	$5.2128E12$	0.0
2nd eq	$-2.7569E11$	$5.2128E12$	0.0	$2.1139E13$
		FE Modal		
8 × 8 mesh	$1.3026E12$	$-8.2691E11$	$5.3487E12$	1.6572
	$-2.7569E11$	$5.3487E12$	4.9357	$2.2128E11$
12 × 12 mesh	$1.2992E12$	$-8.2699E11$	$5.2740E12$	2.2294
	$-2.7566E11$	$5.2740E12$	6.6420	$2.1593E13$
16 × 16 mesh	$1.2981E12$	$-8.2702E11$	$5.2473E12$	54.8464
	$-2.7567E11$	$5.2473E12$	16.4525	$2.1397E13$

Mesh sizes are in a quarter plate, S-S: simply supported.

Table 2 Comparison of rms values of the displacement

Excitation level, g	Exp. results,[23] in.	Modal FEM, in.[a]	% Difference
0.98	0.011 (0.00028 m)	0.012 (0.00030 m)	10.0
1.97	0.019 (0.00048 m)	0.021 (0.00053 m)	10.5
3.91	0.031 (0.00079 m)	0.031 (0.00079 m)	0.00
7.85	0.043 (0.0011 m)	0.044 (0.00112 m)	2.00

[a]FEM = finite element model.

Table 3 Comparison of rms values of the surface strains[23]

Excitation level, g	Exp. results,[23] in.	Modal FEM, in.[a]	% Difference
0.98	40 (1.016 m)	37 (0.939 m)	−7.50
1.97	68 (1.727 m)	65 (1.651 m)	−4.40
3.91	109 (2.769 m)	99 (2.515 m)	−9.20
7.85	161 (4.089 m)	156 (3.962 m)	−3.10

[a]FEM = finite element model.

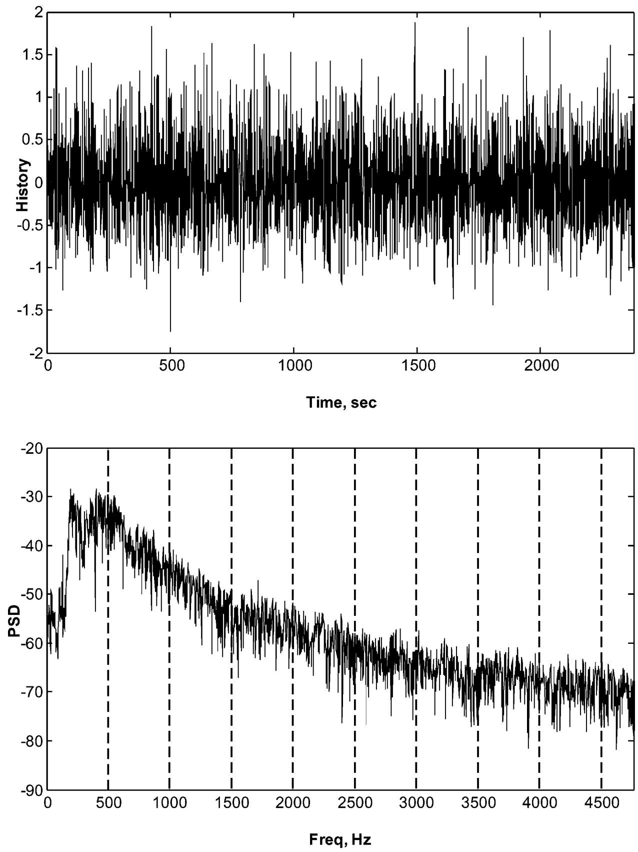

Fig. 3 Time history and PSD of complex process data.

by Brodtkorb et al.[25] The time history and PSD of the complex process are shown in Fig. 3. The RFC method used was defined by Rychlik.[26] For stationary Gaussian narrowbanded process, the closed-form solution for fatigue life is

$$T_{\mathrm{cf}} = \frac{K}{T f_0 \left(\sqrt{2}\sigma_s\right)^{\beta} \Gamma(1+\beta/2)} \tag{10}$$

where f_0 is the mean rate of occurrence of the peaks, $T f_0$ is the expected number of cycles, σ_s is the stress standard deviation, and Γ is the gamma function. The narrowband solution was run for $x(t) = 100\ \sin(t)$ with $t \in [0, 2^8\pi/2]$ yielding to $T_{\mathrm{cf}}^f = 8.036$ and from Eq. (9) $T^f = 7.885$ s, which was less than 2% difference. Verification of the RFC by reproduction of fatigue life of the complex process yielded to identical fatigue lives of 682 years.[25]

Rectangular Isotropic Plates

A simply supported $15 \times 12 \times 0.060$ in. ($38.1 \times 30.48 \times 0.15$ cm) panel with immovable in-plane boundary conditions $u(0, y) = u(a, y) = v(x, 0) = v(x, b) = 0$ is studied in detail. The material properties are the same as of the panel in the preceding section. The plate is modeled with a more than adequate 16×16 mesh (256 BFS elements) in a quarter-plate. The number of active structural node DOF $\{W_b\}$ is 1024. The maximum principal stress σ_1 at the plate center is obtained for fatigue analysis. The lowest six natural frequencies are given in Table 4.

Table 4 Frequencies of $15 \times 12 \times 0.06$-in. isotropic plate

Mode	Exact, Hz	FE, Hz
(1,1)	80.516	80.516
(3,1)	331.818	331.818
(1,3)	473.277	473.292
(3,3)	724.645	724.655
(5,1)	834.618	834.668
(5,3)	1227.730	1227.420

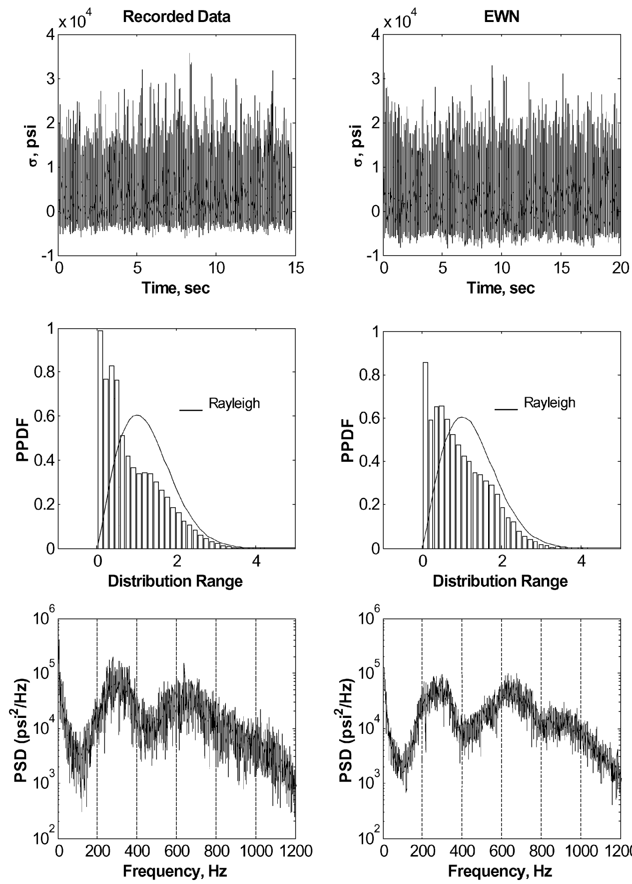

Fig. 4 Time histories, PPDF, and PSD of maximum stress for set 2 and EWN$_2$.

For accurate numerical simulation the time step of integration was studied. First, a time step $\Delta t = 1/8192 = 1.2207 \times 10^{-4}$ s was selected, then the time step was cut into one-half with $\Delta t = 1/(2 \times 8192)$ s, and the maximum deflection time histories for the two integration time steps were compared and found to be exactly identical. To capture the influence of the highest modes to the stress and consequently the fatigue-life estimation, all of the five symmetric modes between zero and the cutoff frequency ($f_c = 1024$ Hz) were considered.

Table 5 gives the statistical moments of the maximum stress response. It showed that the flight data set of nonwhite PSD yielded higher values of rms and mean stress than its corresponding EWN. Differences in rms stresses were about 4–7%, and differences in mean stress were about 16–21%. The high kurtosis value of the recorded data indicated that the PDF deviates largely from a Gaussian distribution, and the lower kurtosis for the EWN showed a closer to Gaussian distribution.

Time history, PPDF, and PSD of maximum stress are shown in Fig. 4. Nonzero mean of the nonlinear stress response was observed from either the time history or the PSD. It is also noticed that the PPDF for nonlinear stress is non-Rayleigh and the flight nonwhite PSD excitation yielded higher PPDF than its corresponding EWN. This is an important discovery because fatigue-life evaluation directly depends on the stress amplitudes to the power of the material

Table 5 RMS and moments of maximum stress for 15 × 12 × 0.06-in. isotropic plate

Calculated	Recorded data	EWN
RMS, psi	5858	5615
Mean, psi	3195	2668
Variance	70.07	70.29
Skewness	1.410	0.939
Kurtosis	2.401	1.160

Mean $= \mu_1$.
Skewness $= \mu_3/\sigma^3$.
Kurtosis $= (\mu_4/\sigma^4) - 3$.
$\mu_k \equiv k$th central moment. $\sigma \equiv$ standard deviation.
variance = sigma^2.

Table 6 Fatigue-life estimate in hours

Panel	15×12×0.06 in.	14 × 10 × 0.06 in.
Recorded	277	7644
EWN	311	9322
% Difference	−12.3	−22.0

property β as expressed in Eq. (8). As a result, higher PPDF of the flight data lead to shorter fatigue life. In addition, similar conclusions as stated in Table 5 could also have been determined from Fig. 4.

Table 6 gives the fatigue-life estimates for material constants $K = 1.52 \times 10^{25}$ and $\beta = 4.8$ estimated from S-N curves for two isotropic plates.

It is readily observed that the stresses for the recorded data of nonwhite PSD yielded much lower fatigue-life estimates than their corresponding EWN. The difference in fatigue life in hours was in the range of 12–27%. These differences arose from the approximations made when evaluating fatigue life of arbitrary loads using the Palgrem–Miner rule. The major sources of approximation are 1) the fact that the PPDF $p(S)$ is non-Rayleigh and has to be evaluated numerically through means of the RFC approach and 2) the damage, Eq. (8), is proportional to the power of β, which implied that the estimate was particularly sensitive to the stress amplitude. To improve fatigue-life estimates, mathematical models should not only account for the non-Rayleigh characteristics of the load/response that affects the stress PPDF $p(S)$, but also the nonwhite load characteristic. This last observation is concluded because knowing that the recorded data and their corresponding EWN have the same PSD (averaged) or acoustic power value the difference in results could only arise from the nonflat (nonwhite) characteristic of the PSD of the recorded data.

Conclusions

This paper addressed the sonic fatigue of isotropic surface panels subjected to a high-intensity flight acoustic pressure fluctuation with nearly non-Gaussian and nonwhite characteristics. A versatile and efficient FE time domain modal formulation was employed to determine the panel nonlinear response with non-Gaussian PSD. The Palmgren–Miner damage theory and the RFC method were used for fatigue estimation of non-Rayleigh stress PPDF. Results showed that the traditional sonic fatigue methods with stationary Gaussian white-noise acoustic pressure were conservative. Recorded flight data of nonwhite PSD gave much shorter fatigue life by 12–27% for the panel studied. More flight data are needed for further investigation and full understanding of nonwhite PSD acoustic pressure on panel response and fatigue life.

Aircraft and spacecraft are designed to perform a variety of missions for different flight regimes. Therefore, response calculations and fatigue-life estimates of the surface panels should reflect the different mission profiles because drastic changes in acoustics conditions could be induced. This would lead to the consideration of nonstationary random process in sonic fatigue design, which has not been addressed in this paper.

Acknowledgments

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